STRANGE QUARK MATTER ATTACHED TO STRING CLOUD IN $f(T)$ GRAVITY

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ABSTRACT

In this paper, Kantowski-Sachs cosmological model with strange quark matter attached to string cloud in the framework of Teleparallel Gravity so called $f(T)$ gravity, where T denotes the torsion scalar has been investigated. The behavior of accelerating universe is discussed by considering exponential $f(T)$ gravity i.e. $f(T)=T\,e^{T}$. The physical behavior of the model has been discussed using some physical quantities.

Keywords: *Kantowski-Sachs model, Quark matter, Cosmic strings,, $f(T)$ gravity.*

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1. Introduction

There has been an increasing interest in modified theories of gravity, in view of the direct evidence of late time acceleration of the universe and the existence of the dark matter and dark energy [1-3]. These observations lead to a matter called dark energy (DE) which has large negative pressure. 73% of the energy of our universe is occupied by DE , 23% is occupied by dark matter whereas the baryon matter occupies only about 4% of the total energy of the universe. Cosmological constant, quintessence, phantom, quintom , chaplygin gas , holographic dark energy etc have been recently proposed as many candidates of dark energy. $f(R)$, $f(G)$, $f(R,T)$, $f(T)$ are the modified theories of gravity which have gained a lot of interest to explain the nature of DE [4]. A particular modified theory of gravity which has attracted the interests of relativists is so-called $f(T)$ teleparallel gravity [5]. In $f(T)$ theory, Weitzenbock connection is used instead of the curvature defined via the Levi-Civita connection in General Relativity. If one chooses to use the Weitzenbock connection, the geometry is flat in the sense that the affine connection has zero Riemann curvature and the field equations are completely described in terms of the torsion tensor. Many relativists [6-8] have discussed the different cosmological models in $f(T)$ theory of gravity.

It is well known that quark-gluon plasma existed during one of the phase transitions of the universe at the early time when the cosmic temperature was $T \approx 200MeV$. Several authors [9-19] investigated quark matter and strange quark matter cosmological models in different contexts which play a significant role in the early stage of evolution of the universe.

In this paper, a research on strange quark matter solutions with $f(T)$ gravitational theory in the presence of Kantowaski-Sachs universe model.

2. $f(T)$ Gravity formalism

We define $f(T)$ theory within Weitzenkock's connection where the line element is described by

$$dS^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \qquad (1)$$

where $g_{\mu\nu}$ are the components of the metric.

The line element (1) can be converted to the Minkowskian description by the transformation called tetrad, as follows

$$dS^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = \eta_{ij}\theta^i\theta^j, \qquad (2)$$

$$dx^{\mu} = e_i^{\mu}\theta^i, \;\; \theta^i = e_{\mu}^i dx^{\mu}, \qquad (3)$$

where $\eta_{ij} = diag[1,-1,-1,-1]$ and $e_i^{\mu} e_{\nu}^{i} = \delta_{\nu}^{\mu}$ or $e_i^{\mu} e_{\mu}^{j} = \delta_i^j$.

The square root of metric determinant is given by $\sqrt{-g} = \det[e_{\mu}^{i}] = e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbock's connection components are defined as

$$\Gamma_{\mu\nu}^{\alpha} = e_i^{\alpha} \partial_{\nu} e_{\mu}^{i} = -e_{\mu}^{i} \partial_{\nu} e_i^{\alpha}. \qquad (4)$$

We define torsion and contorsion tensors as

$$T^{\alpha}{}_{\mu\nu} = e_i^{\alpha} \left(\partial_{\mu} e_{\nu}^{i} - \partial_{\mu} e_{\mu}^{i} \right), \qquad (5)$$

$$K^{\mu\nu}{}_{\alpha} = \left(-\frac{1}{2} \right) \left(T^{\mu\nu}{}_{\alpha} - T^{\nu\mu}{}_{\alpha} - T_{\alpha}^{\mu\nu} \right). \qquad (6)$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor $S_{\alpha}^{\mu\nu}$ from the components of the torsion and contorsion tensors, as

$$S_{\alpha}^{\mu\nu} = \left(\frac{1}{2} \right) \left(K^{\mu\nu}{}_{\alpha} + \delta_{\alpha}^{\mu} T^{\beta\nu}{}_{\beta} - \delta_{\alpha}^{\nu} T_{\beta}^{\beta\mu} \right), \qquad (7)$$

Mathematically the torsion scalar T is defined as

$$T = T^{\alpha}{}_{\mu\nu} S_{\alpha}{}^{\mu\nu} \qquad (8)$$

The modified teleparallel action of $f(T)$ gravity with the matter Lagrangian L_m is expressed
as

$$S = \int e \left[\frac{f(T)}{2k^2} + L_m \right] d^4 x, \qquad (9)$$

where $k^2 = 8\pi G$ is the usual gravitational coupling constant. By varying the action (9) with respect to the tetrads, one gets the following equations of motion

$$S_{\mu}{}^{\nu\rho} \partial_{\rho} T f_{TT} + \left[e^{-1} e_{\mu}^{i} \partial_{\rho} \left(e e_i{}^{\alpha} S_{\alpha}{}^{\nu\rho} \right) + T^{\alpha}{}_{\lambda\mu} S_{\alpha}{}^{\nu\lambda} \right] (1 + f_T) + \frac{1}{4} \delta_{\mu}^{\nu} (T + f) = 4\pi T_{\mu}^{\nu} \qquad (10)$$

where T_{μ}^{ν} is the energy momentum tensor, $f(T)$ denotes an algebraic function of the torsion scalar *T*, $f_T = df(T)/dT$ and $f_{TT} = d^2 f(T)/dT^2$, by setting $f(T) = a_0 =$ constant , the equations of motion (10) are exactly that of the teleparallel theory with a cosmological constant, and this is dynamically equivalent to the General Relativity.

3. Exact Kantowaski-Sachs solution

The Kantowaski-Sachs space-time is

$$ds^2 = dt^2 - A^2 dr^2 - B^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right), \qquad (11)$$

where the metric potentials A and B be the functions of time t only.

The corresponding Torsion scalar is given by

$$T = -2 \left(2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} \right). \qquad (12)$$

The energy momentum tensor for string cloud is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j. \qquad (13)$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s, \qquad (14)$$

where ρ_p is the particle energy density .

Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3}, \qquad (15)$$

where ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c, \qquad (16)$$

where B_c is the vacuum energy density.

But the total pressure is

$$p = p_q - B_c. \qquad (17)$$

In this case from equation (14), we get

$$\rho = \rho_q + \rho_s + B_c. \qquad (18)$$

From equation (13) and (18), we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_s + B_c) u_i u_j - \rho_s x_i x_j, \qquad (19)$$

where u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string.

We have u_i and x_i with satisfying conditions

$$u_i u^i = -x_i x^i = 1 \text{ and } u^i x_i = 0. \qquad (20)$$

We have taken the direction of string along x- axis. Then the components of energy momentum tensor are

$$T_1^1 = \rho_s, T_2^2 = T_3^3 = 0,\ T_4^4 = \rho, \qquad (21)$$

where ρ and p are functions of t only.

Now the field equations for the Kantowaski-Sachs space-time (11) can be written as

$$(T+f)+4(1+f_T)\left\{\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}+4\frac{\dot{B}}{B}\dot{T}f_{TT}=16\pi\rho_s, \qquad (22)$$

$$(T+f)+2(1+f_T)\left\{\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+3\frac{\dot{A}\dot{B}}{AB}\right\}+2\left\{\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right\}\dot{T}f_{TT}=0, \qquad (23)$$

$$(T+f)+4(1+f_T)\left\{\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}=16\pi\rho, \qquad (24)$$

where the dot$(\cdot)$ denotes the derivative with respect to time t.

4. Solution of the field equations

Here we have three differential equations with five unknowns namely A, B, f, ρ_S, ρ. By assuming a form for matter content or relation between metric coefficients, we solve the field equations. A law of variation for Hubble's parameter for spatially homogeneous and isotropic Robertson Walker metric to solve field equations and that yields a constant value of deceleration parameter. The variation of Hubble's parameter as assumed is consistent with observation. Many Cosmologists used the special law of variation of Hubble parameter to solve the field equations.

$$H = la^{-n} = l(AB^2)^{\frac{-n}{3}}, \qquad (25)$$

where $l > 0$, $n \geq 0$ and H is the Hubble parameter defined as

$$H = \frac{\dot{a}}{a}. \qquad (26)$$

An important observational quantity is the deceleration parameter q defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \qquad (27)$$

Using equations (25), (26) and (27), we have

$$q = n-1. \qquad (28)$$

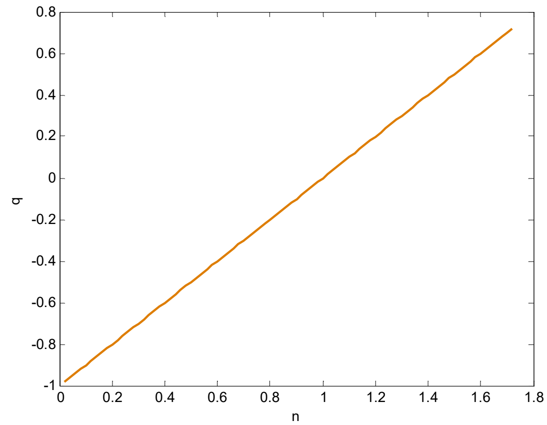

Figure no. 1 . Deceleration Parameter vs n .

The sign of q indicates whether the model inflates or not. The positive of q corresponds to standard decelerating models, whereas the negative sign indicates inflation. The deceleration parameter provides accelerating models for $n < 1$ and decelerating ones $n > 1$. For $n = 1$, $q = 0$,i.e. the expansion of the universe is at constant rate. Recent observations of SN Ia, reveal that the present Universe is accelerating and value of decelerating parameter lies somewhere in the range $-1 < q < 0$. It follows that in the derived model from figure 1, one can choose the values of decelerating parameter consistent with the observations.

Equation (25) can be written as

$$\dot{a}a^{n-1} = l. \qquad (29)$$

Then solving equation (29) , we obtain the law for average scale factor act as

$$a(t) = (nlt + c_1)^{\frac{1}{n}}, n \neq 0, \qquad (30)$$

where c_1 is constant of integration.

Since the field equations (22)-(24) are highly non-linear, a physical assumption that expansion scalar θ is proportional to shear scalar σ which gives

$$A = B^m,\ m \neq 1. \qquad (31)$$

Collins et. al.[20] and Thorne [21] discussed the physical significance of this condition for perfect fluid and barotropic EoS in a more general case .

Using equations (30) and (31), we have

$$A=(nlt+c_1)^{\frac{3}{(1+2m)n}}. \quad (32) \qquad\qquad B=(nlt+c_1)^{\frac{3m}{(1+2m)n}} \quad (33)$$

The metric (11) with the help of equations (32) and (33) can be written as

$$ds^2=dt^2-(nlt+c_1)^{\frac{6}{n(1+2m)}}dr^2-(nlt+c_1)^{\frac{6m}{n(1+2m)}}\left(d\theta^2+\sin^2\theta d\phi^2\right). \quad (34)$$

Here we assume the simplest and commonly considered exponential $f(T)$ gravity i.e.

$$f(T)=T\,e^T. \quad (35)$$

The Torsion scalar T becomes $T=\dfrac{\alpha}{(nlt+c_1)^2}$, (36)

Where $\alpha=\dfrac{-18l^2(m+2)}{(1+2m)^2}$.

Using equation (24), we get the string energy density

$$\rho=\frac{1}{16\pi}\left\{\left[\frac{\alpha_3}{(nlt+c_1)^2}+\frac{\alpha_3}{(nlt+c_1)^2}e^{\frac{\alpha_3}{(nlt+c_1)^2}}\right]+4\left[1+\left(1+\frac{\alpha_3}{(nlt+c_1)^2}\right)e^{\frac{\alpha_3}{(nlt+c_1)^2}}\left(\frac{\alpha_2+6\alpha_1^2}{(nlt+c_1)^2}\right)\right]\right\} \quad (37)$$

where $\alpha_1=\dfrac{3l}{(1+2m)}$, $\alpha_2=\dfrac{9m^2l^2}{(1+2m)^2}$, $\alpha_3=-2\alpha_1^2(m+2)$. (38)

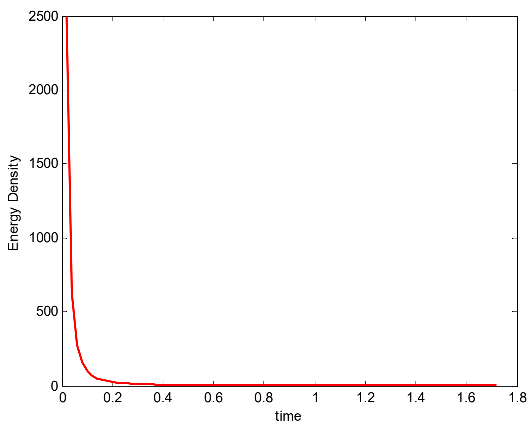

Figure No. 2 Energy Density vs time.

From figure 2 , it is observed that the energy density is a function of time t and always decrease positively with the expansion. From equation (37) it is conclude that at the initial stage of the Universe the energy density approaches to constant value and with the expansion of the Universe it is decreases and at large expansion it is null i.e. $\rho\to 0$. Thus, our derived Universe is free from big rip.

Using equation (22), we get the string tension density

The String particle density is given by

$$\rho_p=\rho-\rho_s$$

$$\rho_p=\frac{1}{16\pi}\left\{\begin{array}{l}4\left[1+\left(1+\dfrac{\alpha_3}{(nlt+c_1)^2}\right)e^{\frac{\alpha_3}{(nlt+c_1)^2}}\left(\dfrac{-\alpha_2+3\alpha_1^2}{(nlt+c_1)^2}\right)\right]\\ -\dfrac{12\alpha_1}{(nlt+c_1)}\left(1+\dfrac{2\alpha_3}{(nlt+c_1)^2}\right)e^{\frac{\alpha_3}{(nlt+c_1)^2}}\left(\dfrac{-2nl\alpha_3}{(nlt+c_1)^3}\right)\end{array}\right\}. \quad (39)$$

The quark energy density is given by

$$\rho_q = \rho - B_c$$

$$\rho_q = \frac{1}{16\pi}\left\{\begin{array}{l}\left[\left[\frac{\alpha_3}{(nlt+c_1)^2} + \frac{\alpha_3}{(nlt+c_1)^2} e^{\frac{\alpha_3}{(nlt+c_1)^2}}\right] + \right. \\ \left. 4\left[1+\left(1+\frac{\alpha_3}{(nlt+c_1)^2}\right)e^{\frac{\alpha_3}{(nlt+c_1)^2}}\left(\frac{\alpha_2+6\alpha_1^2}{(nlt+c_1)^2}\right)\right]\right]\end{array}\right\} - B_c . \quad (40)$$

The quark pressure is given by

$$p_q = \frac{1}{48\pi}\left\{\begin{array}{l}\left[\left[\frac{\alpha_3}{(nlt+c_1)^2} + \frac{\alpha_3}{(nlt+c_1)^2} e^{\frac{\alpha_3}{(nlt+c_1)^2}}\right] + \right. \\ \left. 4\left[1+\left(1+\frac{\alpha_3}{(nlt+c_1)^2}\right)e^{\frac{\alpha_3}{(nlt+c_1)^2}}\left(\frac{\alpha_2+6\alpha_1^2}{(nlt+c_1)^2}\right)\right]\right]\end{array}\right\} - \frac{B_c}{3} . \quad (41)$$

The model (34) has no initial singularity, while the tension density and energy density of the string given by (38) and (40) possess initial singularities. However, as time increases these singularities vanish. At initial epoch ($t=0$) quark pressure and density are infinite, further both decreases as time increases. Therefore, we do not have any exact knowledge of the state of the cosmic strings and quark matter at the initial moment of creation of the universe.

5. Kinematical properties of the Universe

We define the spatial volume V of Kantowaski-Sachs space-time as

$$V = a^3 = AB^2 = (nlt+c_1)^{\frac{3}{n}} . \quad (42)$$

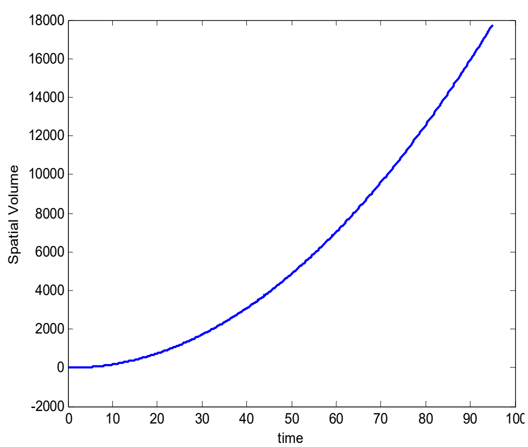

Figure No. 3 Spatial Volume vs time.

The mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = \frac{l}{(nlt+c_1)} . \quad (43)$$

The expansion scalar:

$$\theta = 3H = \frac{3l}{(nlt+c_1)} . \quad (44)$$

The mean anisotropy parameter:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right) = 3 . \quad (45)$$

The shear scalar:

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{9l^2}{(nlt+c_1)^2} . \quad (46)$$

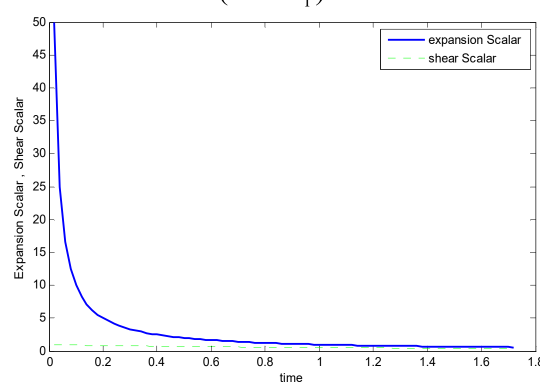

Figure No 4. Expansion Scalar and Shear Scalar vs time .

The spatial volume increases with increase in time representing that the model is expanding as shown in figure 3. It is seen that the average Hubble parameter is a decreasing function of cosmic time. Since the mean anisotropy parameter A_m is constant throughout the evolution of the universe, hence the model is anisotropic throughout the evolution of the universe. The average anisotropy parameter will never vanish in the four dimensional Universe which means that the Kantowaski-Sachs universe will not approach isotropy throughout the evolution. From equations (44) and (46), it is clear that the expansion scalar θ and shear scalar σ are decreasing functions of cosmic time t as shown in figure 4. Also it can be found that $\frac{\sigma}{\theta} \neq 0$. As $t \to \infty$, the scale factor and volume become infinite whereas σ, θ tend to zero. Thus the rate of expansion slows down with the increase of time.

6. Conclusion

In this paper , Kantowaski-Sachs universe with strange quark matter attached to string cloud in $f(T)$ gravity has been investigated. The solution of field equation corresponds to the particular choice of $f(T) = Te^{T}$. It is observed that the scale factors and the spatial volume of the model are zero at $t = t^{*} = \frac{-c_1}{nl}$ and all the remaining parameters are diverging. Therefore, the model has point-type singularity at $t = t^{*}$, which shows that the universe starts evolving with zero volume at $t = t^{*}$ with an infinite rate of expansion . The model is expanding shearing, non-rotating and has no initial singularities. At the initial stage $t \to 0$ the universe has constant energy density but with the expansion of the universe it declines and at large $t \to \infty$ it is null $\rho \to 0$. When, $t \to 0$, $\rho \to \infty$ and when $t \to \infty$, $\rho \to 0$, which indicates that the universe starts with initial (Big-Bang) type of singularity . The decelerating parameter is negative for $n = 0$. Hence, the model of Eq. (34) is inflationary. We hope that our model will be useful in the discussion of structure formation in the early universe and an accelerating expansion of the universe at present.

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